Relations between Length, Elasticity, and Magnetization of Iron and Nickel Wires.

DISSERTATION.

BY

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It has long been known that magnetic metals change their dimensions when magnetized. Begun by Joule[7], the study of these changes has taken the attention of many careful investigators, whose work, especially that of Bidwell[3], has established the following facts with regard to the phenomena observed.

Historical Summary.

Iron.—If subjected to an increasing magnetic field, iron at first elongates in a direction parallel to the direction of its magnetization, more rapidly than in proportion to the magnetizing field, reaches a maximum length, and then contracts nearly in proportion to the magnetizing field, to less than its original length. As the strength of the field is greatly increased, the contraction shows signs of reaching a limit, which was reached by one of Bidwell's[3h] specimens at about $H = 1250$.

The absolute value of the change in length varies with different specimems, but is qualitatively the same in all. If tension[3d, 12] is applied to the iron, the initial elongation is decreased, and a great enough tension destroys it entirely, making the initial phenomenon a contraction.

Hardening[3f, 12] decreases the elongation, and increases the contraction.

Annealing[3f] also diminishes the initial elongation, and increases the final contraction.

Nickel is found, by Bidwell, to contract from the start, approaching a minimum length rather sooner than iron, i. e., at about $H = 900$ c. g. s., for a particular specimen[3h].

Tension[3d] diminishes the contraction in weak fields. In fields of strength greater than 140 or 150, magnetic contraction is increased by tension, up to a critical value, depending on the strength of the field, and diminished by greater tension.

Faults of Previous Work.

In all but one[13b] of the investigations made prior to last winter, a relation between change in length and magnetizing field only

has been given. This fact led last year to the investigation of Dr. More[12] in this laboratory, who got curves between change in length of an iron wire and magnetization.

In order to get the change in length due to the magnetization alone, he considered the metal as under a mechanical stress [6] equal to $\frac{B^2}{8\pi}$, and therefore having suffered a purely mechanical contraction proportional to $\frac{B^2}{8\pi}$. He therefore measured Young's modulus, and the induction B, deduced this mechanical contraction, and added it with positive sign, to his curves between change in length, and I the magnetization. It was known that Young's modulus[4, 15] for iron changed when the iron was magnetized, but Dr. More showed that he could neglect the small correction due to this cause, when the tension in the wire was small. His curves for the wire, under the minimum tension used, are given below. The upper one is intended to show the relation between change in length and magnetization alone, and is obtained by applying the $\frac{B^2}{8\pi}$ correction to the lower one, which is the uncorrected relation as observed. Both curves show that as the magnetization increases, the length increases, at first slowly, then more rapidly, reaching a maximum shortly before saturation, when the wire begins suddenly to contract quite rapidly, and the curve falls in an almost vertical straight line. The $\frac{B^2}{8\pi}$ correction merely raises the curve, without changing its general shape. There is, however, considerable doubt [5b] as to the propriety of this correction.

The careful work of Bidwell principally, and the results to be given presently, show that the contraction of iron, after saturation is reached, is, for a time at least, proportional to the magnetizing force.

Bidwell's experiments with very strong fields [3h] also show that the contraction approaches a limiting value asymptotically. In other words, the retraction curve, after saturation, is first a descending straight line, and then becomes curved with its concave side upward. The curve of $\frac{B^2}{8\pi}$ plotted to H, constantly rises, but

More's Curve.
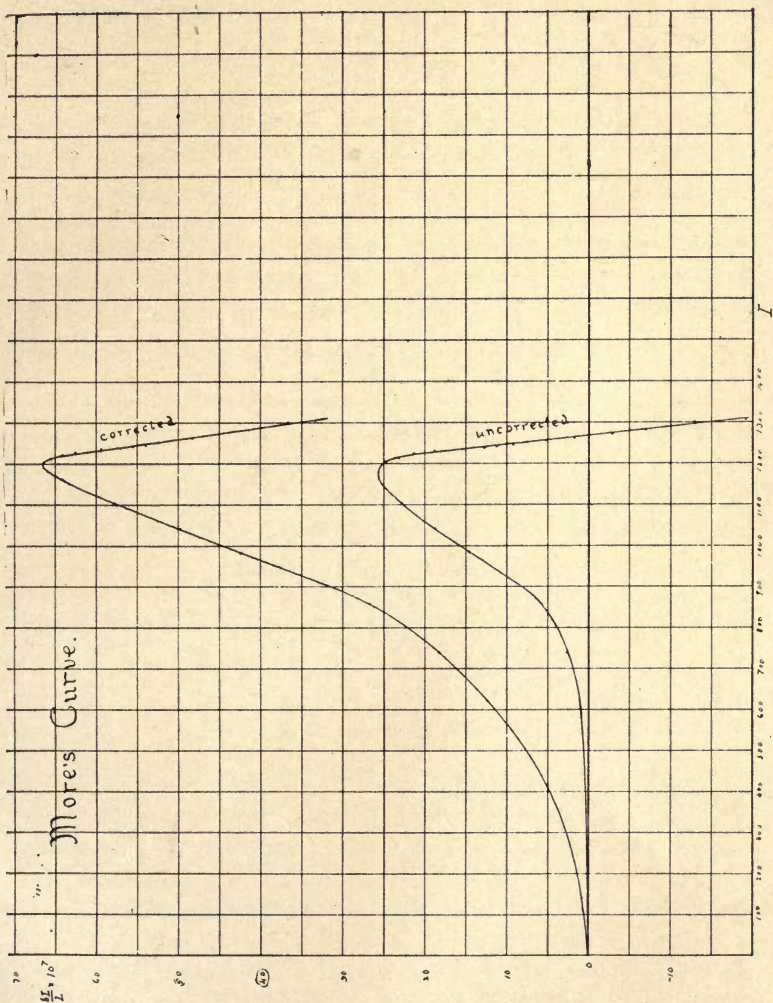
is convex upward where the retraction curve is straight, and concave upward where the inductions are large. If, therefore, that part of the change in length, which is not due to the magnetization, were a contraction proportional to $\frac{B^2}{8\pi}$, the contraction that immediately follows saturation, should be less than proportional to the field; i. e., that part of the contraction curve that immediately follows saturation, should be concave upward, while experiment shows it to be straight. Also, for very strong fields, the rapid increase in $\frac{B^2}{8\pi}$ should make the contraction more rapid than in proportion to the field; i. e., the contraction curve should finally curve downward. The work of Bidwell, already cited, shows the reverse to be the fact. Finally, if the change in length were due to change in I and in $\frac{B^2}{8\pi}$, Dr. More's curve, as corrected, should come to an end at a point given by the maximum elongation as ordinate, and maximum magnetization as abscissa. In reality his curve falls almost perpendicularly just before reaching the point where it should end, showing that after the metal is nearly saturated, the change in length is independent of both the magnetization and of $\frac{B^2}{8\pi}$. It seems, therefore, reasonable to assume that the change in length is also independent of $\frac{B^2}{8\pi}$ before this point is approached.

OBJECT OF INVESTIGATION.

The following investigation was undertaken at the suggestion of Dr. Ames, with the original purpose of studying iron, nickel, cobalt, and bismuth, and in the hope that a tabulation of Young's modulus, along with induction, permeability, field strength, and magnetization, as well as the change in length of each, would show enough connection between some of these quantities, to explain the phenomena. It was, however, impossible to get bismuth and cobalt in the form of wire; hence the investigation is confined to iron and nickel. Also, on account of the many set-backs in getting the apparatus to work satisfactorily in its necessary surroundings, and owing to the short time available, it has been

possible to investigate only one specimen of each metal. This is especially to be regretted, since the results differ unexpectedly, and in an important manner from any given by previous investigators.

Description of Apparatus and Method of Observation.

The apparatus was the same as that used by Dr. More last year, and is shown in figures 1 and 2. The wire was encased in a brass tube a, Fig. 1, open at the upper end and closed at the lower end by a brass plug b. At the top was a bracket c supporting a lever d, and a projecting arm e. The wire experimented on passed up through the brass plug b, in which it was tightly fastened by a set-screw, was held concentric in the tube by a loosely-fitting cork f, in the open end of the tube, carried tightly screwed to it immediately above the cork, a brass hook g, and finally passed to the support above, thus carrying the load of the tube and bracket. The hook g, screwed to the wire, made a knife-edge connection with the short arm of the lever d, which was supported on knife edges by the bracket c.

The projecting arm e of the bracket passed out under and parallel to the long arm of the lever d, and had its extremity bent up, so that this extremity and the upper surface of the end of the lever lay near together and in the same plane when at rest. A small brass table h furnished with three legs made of needle points about 3 mm. apart, was placed on the end of the lever, so that two of its legs rested in a scratch in the lever, while the third leg rested on the raised extremity of the projecting arm of the bracket. This little three-legged table carried a bit of plane glass mirror, in which a vertical scale was observed by means of a telescope. It is evident that any change in the length of the part of the wire between the set-screws in the plug b and the hook g, must change the inclination of the lever d and tilt the mirror at h.

If

L = length of long arm of lever,

l = length of short arm of lever,

d = distance from one leg of brass table to line joining other two bearing on lever end,
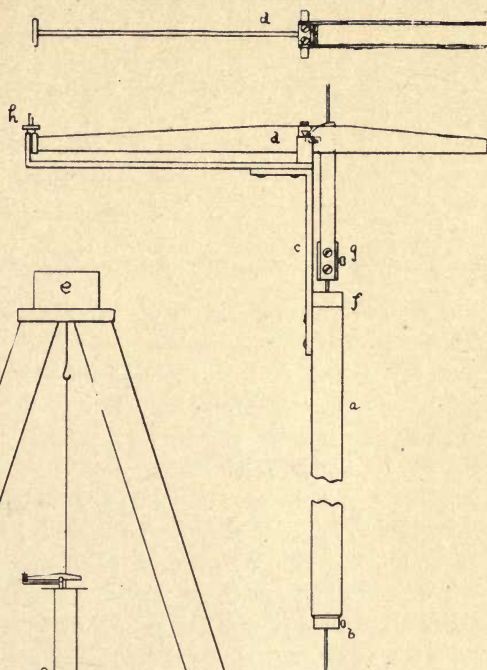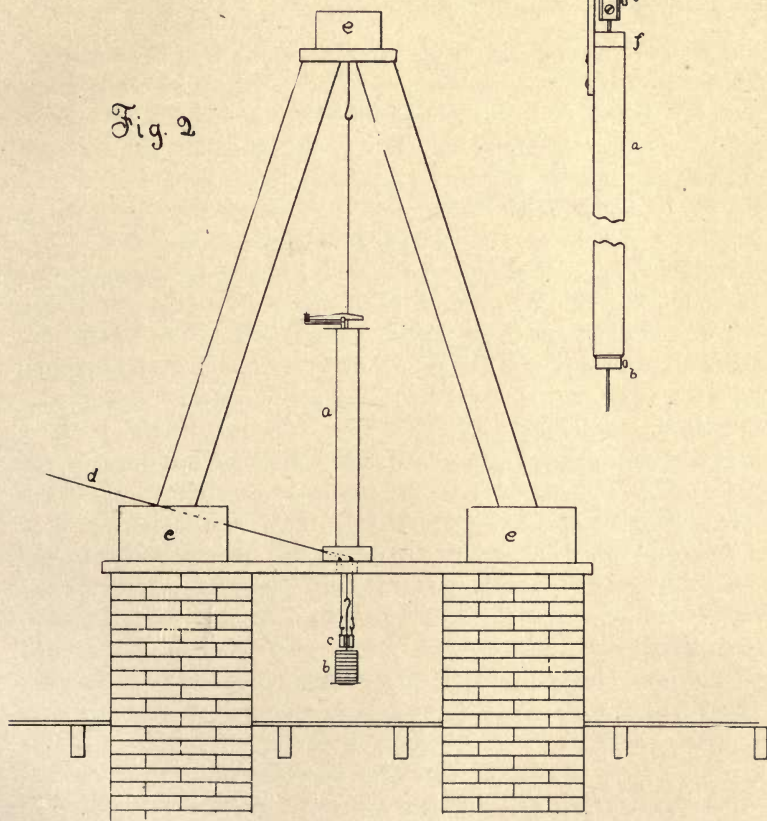
D = distance from scale to mirror,

Fig. 1

Fig. 2

then the multiplying power of the apparatus is

$$\frac{L}{l} \times \frac{D \times 2}{d}.$$

$L = 11.67$ cm. $l = 0.477$ $d = 0.3365$ $D = 147.6$

therefore multiplying power is

$$\frac{11.67 \times 147.6 \times 2}{.477 \times .3365} = 21462.$$

Hence $\dfrac{\text{Scale reading}}{21462}$ gives the actual changes in the length of
the wire. It was possible, under good conditions, to read the
scale to one-tenth of a millimeter, corresponding to an actual
change in length of 0.000000466 cm. The length of the wire
between fastenings = 70 cm. It was therefore possible to de-
tect, with a fair degree of certainty, changes in the unit length
as small as 0.000,000,006,6, or one part in 150,000,000.

The hollow brass tube a, containing the wire, and supported
by it, was placed inside a vertical solenoid a, Fig. 2, considerably
greater in length than the tube itself, thus bringing the set-
screws in b and g, Fig. 1, well within the magnetic poles of the
wire studied. The wire of the solenoid was wound on a brass
tube, inside which another brass tube was placed, thus forming
a jacket between the wire of the coil carrying the magnetizing
current, and the suspended tube containing the specimen studied.
A stream of cold water was kept flowing in this jacket, so that it
took some time for the heating of the current to affect the read-
ing in the telescope. The strength of the field was 45.7 c. g. s.
per ampere. For weak fields the magnetizing current was meas-
ured by a Weston mil-ammeter, and regulated by a water and
copper sulphate resistance. For strong fields, the current was
measured by a Weston ammeter of greater capacity, reading to
hundredths of an ampere, and was regulated by an iron wire re-
sistance. The current was obtained from the storage cells of
the laboratory, and could be kept very constant.

The induction in the wire was measured by the method of in-
creasing reversals.

In the case of the iron, a paper cylinder, wound with two
hundred turns of fine copper wire connected with a Rowland-

d'Arsonval galvanometer, was slipped over the wire, and placed midway between the ends of the tube *a*, Fig. 1.

For the nickel, instead of using the paper cylinder, a thin coating of sealing-wax was first applied, over which were wound four hundred turns of fine silk-covered wire, connected with the galvanometer. After each set of induction measurements, the galvanometer was calibrated by means of a long solenoid wound on a wooden core, and carrying a secondary coil of two hundred turns. The mean area of the secondary coil upon the wire was carefully measured and the section of the wire, whence the inductions were calculated in the usual manner.

The lower end of the wire studied, was provided with a weight-carrier *b*, Fig. 2, so that weights could be added and measurements taken of Young's modulus. The stretching weight, or rider *c*, weighing 46.9 grams, consisted of a piece of brass tubing about 3 cm. in diameter, and the same in length. One side was cut axially, so that it could be applied without removing the weight-carrier. Two loops of copper wire were soldered to opposite sides of this rider, and through these passed light wire hooks suspended by strings which passed through two small pulleys, one on each side of the stretched wire, and about 1.5 cm. from it. Above the pulleys the strings were united into a single one *d*, and carried to the table where the readings were taken, and there fastened. The length of this string was such that when it was fully extended, the rider was supported by the weight-carrier, and the supporting hooks just swung free of the side loops. When it was desired to remove the rider, the string *d* was drawn aside, and held by a tack in the table, which kept the rider freely suspended, about 3 mm. above the weight-carrier.

Great difficulty was experienced in getting trustworthy results from an apparatus of such delicacy. The induction measurements could be made at any time, but the length and modulus measurements had to be made between two and four in the morning, when the traffic of the city was less than at any other time, and even then, the effect of the March winds upon the building was a sore trial to the patience. A very important source of error was, of course, found to be that due to the heating of the current in the coil. It was finally found necessary

to read only instantaneous changes of length, both for the elongation curve, and for the change in Young's modulus.

In measuring Young's modulus there seemed to be an instantaneous elongation upon applying the rider, followed more or less continuously by a slow increase in the reading, showing an apparent viscosity of the metal. On account of the heating, it was impossible to wait for the modulus reading to become constant, but the instantaneous elongation was guessed at as nearly as possible and though, of course, much too small to give the usual value of Young's modulus, and also too irregular to be of any quantitative value, was still near enough what it should have been to show the approximate position of the modulus curve. In any case, whatever the modulus curve may be, the elongation and retraction curve seems to be independent of it, so that accuracy in this respect is not vital. It was also the exception to have the apparatus free from vibrations, though its supports rested in boxes of sawdust e, Fig. 2, and the whole was placed on brick piers built up from the ground. But fortunately the night on which the first iron curve was taken was unusually quiet, and all the conditions most favorable, as the results seem to show, since a smooth curve can be drawn through nearly all the points obtained. The wires were annealed before being set up, and demagnetized by an alternating current before each set of measurements.

The elongation curves were taken as follows : A reading was taken through the telescope, the magnetizing current then increased a small amount, and another reading taken immediately. The process was then repeated without turning off the current, and carried from zero current to between 6 and 7 amperes, or a field of about 300. [The current had to be turned off twice during each set of measurements, to allow a change in the voltage applied, which, on account of high resistance of the solenoid and the low resistance of the rheostats obtainable, had to be varied between 20 and 100 volts. No harmful effects on the results could be detected.] No readings could be repeated, since the alternator supplying the current, used to demagnetize the wire, was not kept running at night. Two sets of curves were taken, the first when the wire carried only the weight of the apparatus, giving a tension of 53 kg. per sq. cm. in the case of the iron, and

of 30 kg. per sq. cm. in case of the nickel, the second when the
load on the wire was increased to 323 kg. per sq. cm. in the iron,
and to 179 kg. per sq. cm. in the nickel. The temperature re-
mained fairly constant somewhere in the neighborhood of 9° C.

RESULTS.

I. *Iron.* Tension = 53 kg. per sq. cm.

The iron wire was one of ordinary commercial iron, number
19 Stubs' gauge, diameter = .106 cm.

The elongations are given in Table I and plotted to H in Plate
I, and to I in Plate V. Values of Young's modulus are given in
Table Ia and plotted to H in Plate I, and to I in Plate V. All
the magnetic quantities are given in Table Ib and plotted to H
in Plate I, and to I in Plate V.

a.) Elongation:

The elongation curve for the iron wire given in Table and
Plate I is the most suggestive of any of those taken. It is
peculiar in showing an initial *contraction.* A glance at the curve
is a much better description than can be put into words. It will
be noticed that the initial contraction, while increasing, increases
more rapidly than in proportion to the magnetizing force, and
reverses quite abruptly at the value of H at which the magnetiza-
tion curve begins to be convex upward, and at which the perme-
ability also changes abruptly from an increase to a decrease.
The length begins at this point to increase quite rapidly, but less
than in proportion to H. After regaining the original length,
the wire continues to elongate to a maximum, and then contracts
again in the way observed by Bidwell, More, and others. This
curve is the last and best of four sets of measurements, which,
for various reasons, had to be discarded, but in all of which the
initial contraction was observed.

This initial contraction can be very satisfactorily explained on
the well-known theory that particles of the metal rotate when
the metal is magnetized.

The last part of this curve seems to show that the wire suffers
a contraction directly proportioned to H after saturation is
reached. It has been assumed, therefore, that the change in

length is equal to the change produced by the magnetization alone, plus a constant times H; or, expressed in symbols,

$$\frac{\delta L}{L} = \left[\frac{\delta L}{L}\right]_{\text{I}} + \gamma H$$

where γ is the slope of the straight part of the curve and is negative. To separate out that part of the elongation due to the magnetization alone we have

$$\left[\frac{\delta L}{L}\right]_{\text{I}} = \frac{\delta L}{L} - \gamma H.$$

If each ordinate of Table I is increased by γH, we obtain Table V, the results of which are plotted in Fig. 3. A relation between elongation and magnetization is taken from this curve as in Table VI and plotted in Fig. 4.

b.) Change in Young's modulus.

The points of the curve are seen to be a good deal scattered, but serve well enough the purposes of this paper. There is a total increase of about 30%, the maximum being reached at about saturation, followed by a decrease to nearly the original value at $H = 100$. A comparison between the curves for Young's modulus and for elongation, seems to show that they are not connected.

II. *Iron.* Tension = 323 kg. per sq. cm. Same wire as in I.

The elongations are given in Table II and plotted to H in Plate II.

Values of Young's modulus are given in Table II*a* and plotted to H in Plate II.

All the magnetic quantities are given in Table II*b* and plotted to H in Plate II.

a.) Elongation.

This elongation curve adds nothing, except that a small increase in the tension of the wire almost annuls the initial contraction, increases the elongation and final contraction.

b.) Young's modulus.

This, as might be expected, shows a greater change under the increased load, the percentage increase being about the same, but the final value only about 80% of the original value.

III. *Nickel.* Tension = 30 kg. per sq. cm.

Wire obtained from Eimer & Amend, of New York. Number 15 American gauge, diameter = .143 cm.

Elongations are given in Table III and plotted to H in Plate III, and to I in Plate VI. Values of Young's modulus are given in Table IIIa and plotted to H in Plate III, and to I in Plate VI. All the magnetic quantities are given in Table IIIb and plotted to H in Plate III, and to I in Plate VI.

a.) Elongation:

The wire increased in length, at first more rapidly, then more slowly than in proportion to the magnetizing field, reaching a maximum length just before the point of maximum permeability. Contraction then began, soon bringing the wire to its original length, and continued much more rapidly than in the iron, and in proportion to the magnetizing force, until saturation was approached, when the decrease in length began to be less than in proportion to the magnetizing force, and so continued till the end of the experiment at about $H = 300$.

The curve, however, offers no opportunity for the separation of the part of the elongation due to magnetization alone, from that part due to the magnetizing force.

b.) Young's modulus.

The modulus curve shows an increase of about 26% with the maximum nearly coincident with maximum elongation, followed by a gradual decrease to about 80% of its original value. This curve, though it suggests that the change in Young's modulus may be due to the change in the molecular arrangement, nevertheless shows that the change in length is not at all due to the change in the modulus. For while an increase in the modulus might cause contraction, it certainly could not cause expansion, and similarly, a decrease in Young's modulus might cause expansion, but not contraction.

IV. *Nickel.* Tension = 179 kg. per sq. cm.

Same wire as in III.

Elongations are given in Table IV and plotted to H in Plate IV.

Values of Young's modulus are given in Table IVa and plotted to H in Plate IV.

All the magnetic quantities are given in Table IV*b* and plotted to *H* in Plate IV.

a.) Elongation.

The small increase in the tension of the wire seems to cause a decrease in the initial expansion, quite analogous to the decrease in the initial contraction in the case of iron. Bidwell's [3d] work on the effect of tension in the contraction of nickel shows that in strong fields, as the tension is increased, the contraction at first increases and then decreases, which is analogous to the increase and subsequent decrease in the elongation of iron produced by increasing tension.

b.) Young's modulus.

The modulus curve does not seem to be materially altered by the increased tension.

DISCUSSION OF RESULTS.

I. *Iron.* The curves of Plate I and Fig. 3 afford a striking verification of Weber's [10] theory that the molecules of iron rotate when magnetized, which has been elaborated by Maxwell and Ewing, and indicate that this rotation is the common cause of both the magnetization and elongation.

Thus, suppose the metal to be made up of or to contain small particles of oblong shape, and magnetized in the direction of their length. Take any plane section of the wire at right angles to its axis: If there is no magnetization of the iron, the particles in this section may be supposed to be pointing in all directions, thus magnetically neutralizing one another.

Now, suppose that all these particles, with their directions kept unchanged, are moved so that their south poles coincide. There will thus be formed a spherical pencil of rays, each ray a small magnet, with the south poles at the center and the north poles in the surface, as in the figure, where all the south poles are at *o* and the north poles as at *a*, *b*, *c*, *d*. If a magnetizing force is now applied in the direction of the arrow, the particles, radii in the figure, will
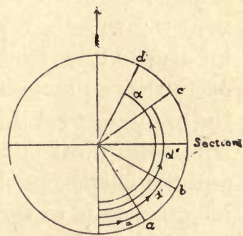
rotate, as shown. If the ends of the particles are in some way connected to the metal, this rotation should produce a change in the thickness of the section proportional to the sum of the changes in the cosines of the angles made by the particles with the axis.

For a weak field the rotations should be small, but those particles that originally pointed downward, as $o\,a$, $o\,b$, should turn more than those in the upper hemisphere, since in the lower hemisphere the turning moments increase as the particles turn, while in the upper hemisphere they decrease.

The change in the arithmetical value of the cosines in the lower hemisphere is negative, but of those in the upper positive. Hence a small magnetizing force should produce a contraction of the wire. Also, on account of the increasing moments in the lower hemisphere, the wire should contract more rapidly than in proportion to the magnetizing force, until what might be called the "average particle," has passed the horizontal position, when the sign of the changes in length should abruptly reverse, and the wire elongate, gradually approaching a maximum length which should be reached at saturation, when the particles are supposed to have become parallel. If the above is correct, it should be possible to find a connection between the elongation and magnetization curves. This cannot be done, of course, without knowing something not only of the shape of the particles, but, also, of the law by which they resist turning. But a comparison between the two curves can be made thus: Consider the part of the magnetization curve that is concave upward, $i.\ e.$, from $H = 0$ to $H = 2.45$. It is evident that before that magnetization is reached that corresponds to $H = 2.45$, I increases more rapidly than in proportion to H. Now, in the theory given above, each particle is supposed to turn under the influence of a magnetizing force and contribute to the total magnetization an amount proportional to the change in the cosine of the angle formed by it with the perpendicular. If the particle is below the horizontal, it will turn under an increasing moment, and the change in magnetization, due to it, will be greater than in proportion to the turning force. If the particle is above the horizontal, the change in magnetization due to it will be less than in proportion to the turning force. Conversely, since the magnetization up to $H = 2.45$ increases more rapidly than in propor-

tion to the magnetizing force, the average motion of the particles must be below the horizontal. But the contribution of each particle to the change in length, is also given by the change in the cosine of its angle, is negative if the particle is below, positive if above the horizontal. Hence the magnetization curve, if plotted with its sign changed, as far as $H = 2.45$ should resemble the elongation curve up to that point. Now consider the magnetization curve between $H = 2.45$ and $H = 2.93$. This part is quite straight, which shows that the change in magnetization is proportional to the force, *i. e.*, that the average motion of the particles is in the horizontal. But here the positive changes in the cosine just balance the negative changes, and we should get no change in length. Continuing from $H = 2.93$ the magnetization curve is convex upward, showing that the average motion of the particles is above the horizontal. Hence the elongation curve and magnetization curve should be similar beyond $H = 2.93$.

If $I' =$ the value of I at $H = 2.45$, and I'' the value of I at $H = 2.93$, the comparison curve already carried to $H = 2.93$ might be continued from this point by plotting ordinates equal to $I - I'' - I'$. Now, the elongation, when the average motion is above the horizontal, should be very much greater than the contraction taking place when the average motion is below the horizontal, since in the former case a majority of the particles work together. The elongation ordinates should be therefore probably anywhere from two to five or six times greater than the contraction ordinates. If they are taken twice as great, and the comparison curve is continued from $H = 2.93$, by plotting ordinates given by $2 (I - I'') - I'$, we get the curve given in Table VII and Fig. 5. The similarity between this curve and the elongation curve is evident enough to make it seem probable that if we knew the law by which the particles resisted turning, we could, without any guess-work, construct from the magnetization curve one that would approximate very closely to the elongation curve. It seems, therefore, that the elongation curve of Plate I proves quite clearly that the initial contraction, elongation, and magnetization are all due to the same cause, namely, the actual rotation of particles in the metal.

The straight part of the curve does not seem to suggest any specific positive conclusion, except that already given, that the

part of the contraction not due to the magnetization is proportional to H up to a certain limit. Some negative conclusions, however, are quite evident. The strain in the metal produced by the magnetic field, after saturation, cannot be like an ordinary mechanical one, for since, within the elastic limit, strains are proportional to stresses, the curve should continue straight for the strongest fields attainable with a solenoid, or if it curved at all it should be concave downward. Bidwell has shown[3h], however, that after a certain strength of field has been passed, the contraction is less than in proportion to the field, and for the specimen he used, ceased entirely at about $H = 1250$, the curve beyond this point being horizontal. To conclude, therefore, the initial contraction and subsequent elongation may be explained as due to the rotation of particles in the iron, and the final contraction, whatever may be its cause, is not an ordinary mechanical strain, and is not, as one would expect from the equations of Maxwell, and from experiments on magnetic tractive force[6], proportional to $\dfrac{B^2}{8\pi}$.

The effects of a small increase in tension, upon the elongation due to magnetization, as shown in Plate II, namely, decrease in the initial contraction and increase in the elongation, are what one would expect, if there is any looseness of the particles, as must be the case, if they can rotate. It is, however, hard to see why tension should increase the final contraction.

II. *Nickel:*

The explanation of the elongation curves for nickel, Plates III and IV, is not so easy as in the case of the iron.

If the particles were flattened instead of elongated, in the direction of their magnetic axes, their rotation would combine with the contraction proportional to the field to make up the curve as far as $H = 90$. Since, however, the ultimate effect of the rotation would be contraction, and what might be called the magnetic elastic limit, is approached so early, it would be impossible to separate out the change due to the rotation alone, for comparison with the magnetization curve.

The effects of a small increase in the tension of the wire, namely, decrease in the initial expansion, and, according to Bid-

well, increase in the final contraction, seem to complete the analogy between the phenomena shown by iron and nickel, provided the particles of the latter are flattened in the direction of their magnetic axes.

DISCREPANCIES BETWEEN THESE RESULTS AND THOSE OF OTHERS.

In the case of iron, it has been impossible to find in the work of other investigators, any mention of an initial contraction, like that shown in Table and Plate I.

Similarly in the case of nickel, it has been impossible to find any record of an initial elongation like that shown in Table and Plate III. In the figures given by More, in his work on iron, the weakest field used is 4.6, at which strength of field, the iron wire of Plate I has more than regained its original length. It is, however, hard to see why these initial effects are not recorded in the extensive work of Bidwell. Many of his tables show initial fields weak enough to produce the initial contraction in iron, and initial expansion of nickel.

His remarks on the peculiarities shown by annealed specimens [3f] of iron, suggest that the annealing to which the specimens examined in this paper were subjected, put them in a condition to show the unusual initial effects, and that if Bidwell had given more attention to his annealed specimens, and examined them with weaker fields and lighter loads, the initial contraction of iron, and expansion of nickel, would not have escaped his notice.

The results for iron, shown in Tables and Plate I, are also shown in Plate V, but plotted to magnetization instead of magnetizing field. The great differences between the elongation curve and the uncorrected curve of Dr. More, already given, are obvious, and illustrate what variations may be shown by two different specimens, though studied with the same apparatus. Plate V shows better, perhaps, than Plate I how greatly the change in length is affected by the magnetizing field, after the change due to the magnetization has practically ceased. Plate VI shows the results for nickel of Tables and Plate III, plotted to I instead of H, but does not seem to throw any additional light upon the matter.

SUMMARY OF RESULTS.

Iron. Phenomena observed.

1. The wire showed an initial contraction when magnetized, contracting more rapidly than in proportion to the magnetizing field. At about the point where the magnetization increases most rapidly and the permeability is greatest, this contraction ceased and the wire began to expand more slowly than in proportion to the magnetizing field, till a maximum length was reached in the neighborhood of saturation. The wire then contracted in direct proportion to the magnetizing field, up to $H = 250$, where the experiment was brought to an end.

2. The instantaneous modulus of elasticity was found to increase over 30%, reaching a maximum at saturation, but decreased again to nearly its original value at about $H = 300$.

3. A small increase in the tension of the wire reduced the initial contraction, did not appreciably change the percentage increase in the modulus, but the final decrease in the modulus left it at only about 80% of its original value.

Conclusions.

1. The initial decrease in the length of the wire and the elongation, are explained as due to the rotation of particles in the metal.

2. The final contraction is not an ordinary mechanical strain, is not proportional to $\frac{B^2}{8\pi}$, but is proportional to the magnetizing field.

3. The change in the modulus seems to have nothing to do with the change in length.

Nickel. Phenomena observed.

1. The nickel wire, when magnetized, increased in length, at first more rapidly, then more slowly than in proportion to the magnetizing field, reaching a maximum length just before the point where the permeability was greatest. Contraction then began, soon bringing the wire to its original length, and con-

tinued much more rapidly than in the iron, and in proportion to the magnetizing force, until saturation was approached, when the decrease in length began to be less than in proportion to the magnetizing force, and so continued till the experiment was ended at about $H = 300$.

2. The instantaneous modulus of elasticity increased about 26%, reaching a maximum at about the same field as that of the maximum elongation, and then fell to 80% of its original value at about $H = 300$.

3. A small increase in the tension of the wire reduced the original expansion, the increase in the modulus was about 23%, and its final value 88% of its original value.

Conclusions.

1. The results for nickel are not so easily explained as those for iron. They do not, however, conflict at all with those for iron, but confirm, though not very definitely, the conclusions drawn therefrom.

2. The change in the modulus does not seem to have anything to do with the change in length.

I ought to say that the conclusions drawn in this paper are entirely my own, and must not be taken as involving the opinion of any one connected with this University.

In closing, I would express most hearty thanks to Dr. Ames for his kindness and help throughout this work, and to Prof. Rowland for his consideration and suggestions.

<div style="text-align: right">EDSON F. GALLAUDET.</div>

Johns Hopkins University, *May*, 1896.

Tension in wire = 53 kg. per sq. cm.

H	$\frac{\delta L}{L} \times 10^8$	H	$\frac{\delta L}{L} \times 10^8$		$\frac{\delta L}{L} \times 10^8$
1.1	— 3.99	11.70	57.91	66.29	5.99
1.64	— 6.66	13.58	58.57	82.74	— 20.63
2.24	— 10.65	14.52	58.57	94.63	— 43.93
3.02	— 21.30	17.33	58.57	105.60	— 63.23
3.34	— 20.63	18.51	58.57	118.86	— 89.86
3.66	— 13.98	23.31	58.57	136.23	— 124.47
3.93	— 7.32	27.84	57.24	158.17	— 171.06
4.20	— .67	33.00	54.58	191.10	— 236.3
6.35	+ 32.62	36.75	51.25	224.46	— 299.53
7.22	39.27	41.33	45.93	251.43	— 352.78
8.91	47.92	46.95	37.27		
10.29	53.92	54.86	25.96		

TABLE I*a.*

Values of the modulus of elasticity.

H	$\frac{PL}{al} \times 10^{12} = M$	H	$\frac{PL}{al} \times 10^{12} = M$	H	$\frac{PL}{al} \times 10^{12} = M$
2.89	2.79	26.57	3.37	128.00	3.72
4.02	2.79	41.28	3.40	150.04	3.80
4.98	2.84	47.13	3.47	165.49	3.64
6.81	2.90	56.14	3.52	179.66	3.55
9.05	2.96	65.56	3.62	205.26	3.52
11.34	3.12	73.60	3.62	211.20	3.35
13.81	3.23	84.12	3.72	256.00	3.34
15.91	3.24	91.89	3.65	301.72	3.07
18.38	3.24	109.26	3.62		
21.58	3.37	119.77	3.74		

TABLE I*b*.

H	B	μ	I	H	B	μ	I
.9	193	214	15.3	22.83	15351	672	1220
1.2	268	223	21	28.89	15565	539	1236
1.5	367	245	29	34.41	15840	460	1258
1.8	516	287	41	38.4	16002	417	1270
2	663	331	53	41.37	16021	387	1272
2.2	900	409	71	47.64	16175	340	1283
2.3	1087	473	86	52.71	16334	310	1296
2.4	1446	603	115	60.44	16514	273	1309
2.51	2835	1129	225	64.73	16586	256	1315
2.7	5554	2057	442	68.57	16630	243	1319
2.826	7571	2679	602	74.97	16873	225	1337
3.2	10641	3325	847	81.14	16880	208	1337
4.43	12437	2808	989	89.60	17115	191	1335
5.49	13288	2421	1057	99.66	17374	174	1375
6.86	13815	2014	1099	125.26	17841	142	1410
8.14	14058	1727	1118	151.09	18280	121	1443
10.06	14393	1431	1145	178.29	18329	103	1444
12.43	14651	1179	1165	205.03	18793	92	1479
15.54	14905	959	1185	228.57	18942	83	1489
19.02	15217	800	1209	299.44	19571	65	1534

TABLE II.

Wire under tension of 323 kg. per sq. cm.

H	$\frac{\delta L}{L} \times 10^8$	H	$\frac{\delta L}{L} \times 10^8$	H	$\frac{\delta L}{L} \times 10^8$
1.05	— .67	10.93	72.55	69.94	— 17.97
1.23	— .67	12.75	72.55	80.46	— 38.61
1.51	— 1.33	14.31	75.21	100.12	— 79.21
1.87	— 1.33	16.09	73.88	110.17	— 103.84
2.51	+ 32.62	18.70	64.56	124.80	— 137.12
3.11	59.90	...	57.91	142.63	— 181.71
3.70	65.23	29.53	56.58	166.40	— 235.62
4.30	69.89	35.89	44.60	201.15	— 312.17
5.94	71.22	40.00	41.27	246.86	— 392.05
6.35	69.89	45.72	28.62	269.72	— 445.30
7.45	69.89	53.76	17.31		
9.14	72.55	64.41	— 3.99		

TABLE IIa.

Values of modulus of elasticity.

H	$\frac{PL}{al} \times 10^{12} = M$	H	$\frac{PL}{al} \times 10^{12} = M$	H	$\frac{PL}{al} \times 10^{12} = M$
2.88	7.74	33.28	10.47	113.38	11.56
3.66	9.44	36.98	10.33	121.60	10.46
5.03	9.68	44.25	11.06	131.66	10.06
7.45	9.33	52.62	10.76	143.09	10.06
9.13	9.00	58.47	11.39	157.26	8.60
12.02	9.68	65.24	11.91	174.64	8.60
15.04	9.68	68.57	11.73	197.49	7 67
18.38	9.68	79.09	11.91	211.66	7.74
23.64	9.68	85.94	12.49	237.72	8.15
27.93	9.80	103.05	12.10	301.73	7.45

TABLE IIb.

H	B	μ	I	H	B	μ	I
.8	168	210	13.3	12.71	14699	1157	1169
1	214	214	17	15.77	14892	944	1184
1.3	293	225	23	20.43	15197	744	1208
1.5	353	235	28	27.02	15552	576	1235
1.7	429	252	34	31.91	15680	491	1245
1.9	524	276	42	39.27	15799	402	1254
2.1	666	317	53	48.92	16174	331	1283
2.3	962	418	76	55.43	16267	293	1290
2.42	1824	754	145	66.04	16433	249	1302
2.70	5952	2204	473	75.43	16545	219	1311
2.88	8106	2815	645	91.43	16954	185	1342
3.11	10858	3491	864	116.57	17417	149	1377
3.22	11901	3696	947	141.72	17852	126	1409
4.57	13093	2865	1042	172.57	18148	105	1430
6.33	13905	2197	1106	215.32	18607	86	1464
7.84	14162	1806	1126	222.86	18872	85	1484
10.38	14561	1403	1158	297.15	19420	65	1522

TABLE III.—*Nickel.*

Tension in wire = 30 kg. per sq. cm.

H	$\frac{\delta L}{L} \times 10^8$	H	$\frac{\delta L}{L} \times 10^8$	H	$\frac{\delta L}{L} \times 10^8$
.59	.67	11.02	46.59	64.37	— 679.59
.78	.67	12.89	51.92	70.40	— 779.44
1.05	1.33	14.22	52.58	87.77	— 992.43
1.37	2.00	16.23	48.59	101.94	— 1152.2
2.10	6.00	18.83	35.28	119.32	— 1333.2
3.38	12.65	22.26	1.33	146.74	— 1660.7
4.57	18.64	27.29	— 60.57	170.52	— 1848.4
5.12	19.97	32.50	— 133.79	206.17	— 2061.4
5.48	21.30	35.89	— 185.71	242.29	— 2234.5
6.35	26.63	42.97	— 320.16	315.44	— 2488.7
7.54	33.28	49.69	— 433.32		
9.23	41.27	58.52	— 566.44		

TABLE III*a.*

Values of the modulus.

H	$\frac{PL}{al} \times 10^{11} = M$	H	$\frac{PL}{al} \times 10^{11} = M$	H	$\frac{PL}{al} \times 10^{11} = M$
0	6.92	13.71	8.65	100.11	7.50
.23	7.40	18.20	8.72	116.57	7.80
1.01	7.53	23.68	8.55	139.89	7.36
1.69	7.87	32.18	8.40	175.09	5.94
2.38	7.97	39.41	8.33	211.20	6.09
3.29	8.17	45.53	8.22	233.15	6.04
4.21	8.28	52.94	8.36	278.86	5.76
5.99	8.43	63.96	8.25	329.15	5.64
7.41	8.51	69.94	8.17		
10.15	8.65	84.57	7.72		

TABLE IIIb.

H	B	μ	I	H	B	μ	I
1	24	24	1,8	31.91	2711	85	213
2	48	24	3.6	37.58	2997	80	236
3	74	25	5.6	45.99	3362	73	264
4	105	26	8	54.86	3662	67	278
5	149	30	11.4	66.88	3969	59	311
5.5	174	32	13.4	77.26	4245	55	332
6.13	246	40	19	97.83	4556	47	355
6.99	293	42	23	129.37	4942	38	383
8.23	405	49	32	142.4	5064	36	392
9.28	485	52	38	173.49	5305	31	408
10.65	604	57	47	212.12	5495	26	420
15.22	1220	80	96	221.26	5635	25	431
20.8	1882	91	148	242.29	5633	23	429
25.69	2313	90	182	290.29	5787	20	437

TABLE IV.

Wire under tension of 179 kg. per sq. cm.

H	$\dfrac{\delta L}{L} \times 10^8$	H	$\dfrac{\delta L}{L} \times 10^8$	H	$\dfrac{\delta L}{L} \times 10^8$
.23	.67	9.10	23.96	67.89	— 757.5
.73	.67	12.98	19.30	87.77	— 1052.3
1.14	2.00	16.32	4.66	121.14	— 1398.5
1.83	6.66	20.62	— 34.61	148.57	— 1667.4
2.83	8.65	27.29	— 114.49	181.95	— 1940.3
3.61	10.65	30.03	— 170.4	212.12	— 2149.9
4.53	12.65	37.17	— 278.9	242.29	— 2372.3
5.99	17.97	45.30	— 397.4	288	— 2579.9
7.04	21.30	57.56	— 579.1		

TABLE IV*a*.

Values of the modulus.

H	$\frac{PL}{al} \times 10^{12} = M$	H	$\frac{PL}{al} \times 10^{12} = M$	H	$\frac{PL}{al} \times 10^{12} = M$
0	3.38	4.98	4.09	66.97	3.73
.46	3.49	6.17	4.09	78.17	3.55
.91	3.67	7.86	4.05	86.40	3.55
1.37	3.87	9.28	4.21	109.26	3.11
1.83	3.97	11.89	4.17	134.86	3.30
2.29	3.90	14.54	4.29	156.35	3.11
2.74	4.01	18.24	4.52	192.00	3.30
3.20	4.09	30.04	4.09	223.55	3.30
3.66	4.13	40.46	3.90	274.29	2.84
4.11	4.13	53.21	3.55		

TABLE IV*b*.

H	B	μ	I	H	B	μ	I
2	52	26	4	18.29	1451	79	114
3	80	27	6.1	22.86	1866	82	147
4	117	29	9	32.69	2479	76	195
5	159	32	12.2	40.69	2860	70	224
6	207	35	16	49.69	3209	65	251
6.86	253	37	19.6	59.34	3501	59	274
7.77	333	43	26	73.6	3887	53	303
8.69	397	46	31	87.32	4185	48	326
10.06	484	48	38	97.37	4357	45	339
10.97	564	51	44	116.34	4530	39	351
11.89	676	57	53	143.09	4945	35	382
12.8	788	62	62	192.46	5330	28	409
14.63	1036	71	81	240	5564	23	424
16	1196	75	94	333.72	5904	18	443

TABLE V.

Elongations of Table I due to magnetization alone. $\gamma = 1.98$.

H	$\frac{\delta L}{L} \times 10^8$	$\frac{\delta L}{L} \times 10^8 + \gamma H$	H	$\frac{\delta L}{L} \times 10^8$	$\frac{\delta L}{L} \times 10^8 + \gamma H$
1.1	— 3.99	— 1.81	23.21	58.57	104.80
1.65	— 6.66	— 3.40	27.84	57.24	112.45
2.24	— 10.65	— 6.21	33.00	54.58	120.03
3.02	— 21.30	— 15.31	36.75	51.25	124.13
3.34	— 20.63	— 14.01	41.33	45.93	127.90
3.66	— 13.98	— 6.72	46.95	37.27	130.38
3.93	— 7.32	+ .47	54.86	25.96	134.76
4.20	— .67	+ 7.66	66.29	5.99	137.46
6.35	+ 32.62	45.21	82.74	— 20.63	143.46
7.22	39.27	53.59	94.63	— 43.93	143.75
8.91	47.92	65.59	105.60	— 63.23	146.20
10.29	53.92	74.33	118.86	— 89.86	145.87
11.70	57.91	81.11	136.23	— 124.47	145.71
13.58	58.57	85.50	158.17	— 171.06	142.63
14.52	58.57	87.37	191.10	— 236.30	142.70
17.33	58.57	92.94	224.46	— 299.53	145.63
18.51	58.57	95.28	251.43	— 352.78	145.86

TABLE VI.

I	$\frac{\delta L}{L} \times 10^8$	I	$\frac{\delta L}{L} \times 10^8$	I	$\frac{\delta L}{L} \times 10^8$
20	— 2	560	— 12	1180	88
40	— 4	710	— 14	1220	103.2
70	— 6	770	— 15.5	1258	121.6
100	— 7.2	840	— 16.8	1296	134.2
130	— 7.6	870	— 14.7	1337	141.1
160	— 7.8	910	— 8	1360	144
190	— 8	950	0	1400	144.2
220	— 8.4	1000	14	1534	144.2
330	— 9.4	1060	34		
450	— 10.5	1100	52		

TABLE VII.

H	$a = -I$	H	$a = 2\,(I - I'') - I'$
1	— 18	7	650
1.5	— 24	11.8	770
2	— 53	16.5	830
2.3	— 86	19	868
2.45	— 150	40	990
...	$a = 2(I - I'') - I'$	100	1180
2.93	— 150	180	1350
3.04	10		
3.2	130		
3.6	270		$I' = 150$
4.5	450		$I'' = 700$

PLATES AND DIAGRAMS.

Plate 1. Iron. Curves of change in length, magnetization, permeability, induction, and Young's modulus, all plotted to H.
Tensile stress = 53 kg. per sq. cm.

Plate I. Enlarged. Enlargement of first part of elongation and magnetization curves of Plate I.

Plate II. Iron. Curves of change in length, magnetization, permeability, induction, and Young's modulus, plotted to H.
Tensile stress = 323 kg. per sq. cm.

Plate III. Nickel. Curves of change in length, magnetization, induction, permeability, and Young's modulus, plotted to H.
Tensile. stress = 30 kg. per sq. cm.

Plate IV. Nickel. Curves of change in length, magnetization, induction, permeability, and Young's modulus, plotted to H.
Tensile stress = 179 kg. per sq. cm.

Plate V. Iron. Curves of change in length, permeability, Young's modulus, and magnetizing field, plotted to I.
Tensile stress = 53 kg. per sq. cm.

Plate VI. Nickel. Curves of change in length, permeability, Young's modulus, and magnetizing field, plotted to I.
Tensile stress = 30 kg. per sq. cm.

Figure 3. Iron. Curve of change in length due to magnetization alone, plotted to H.
Tensile stress = 53 kg. per sq. cm. First part enlarged.

Figure 4. Iron. Curve of change in length due to magnetization alone, plotted to magnetization.
Tensile stress = 53 kg. per sq. cm.

Figure 5. Comparison curve, constructed from magnetization curve of Plate I, and plotted to H, showing similarity to elongatian curve of Fig. 3.
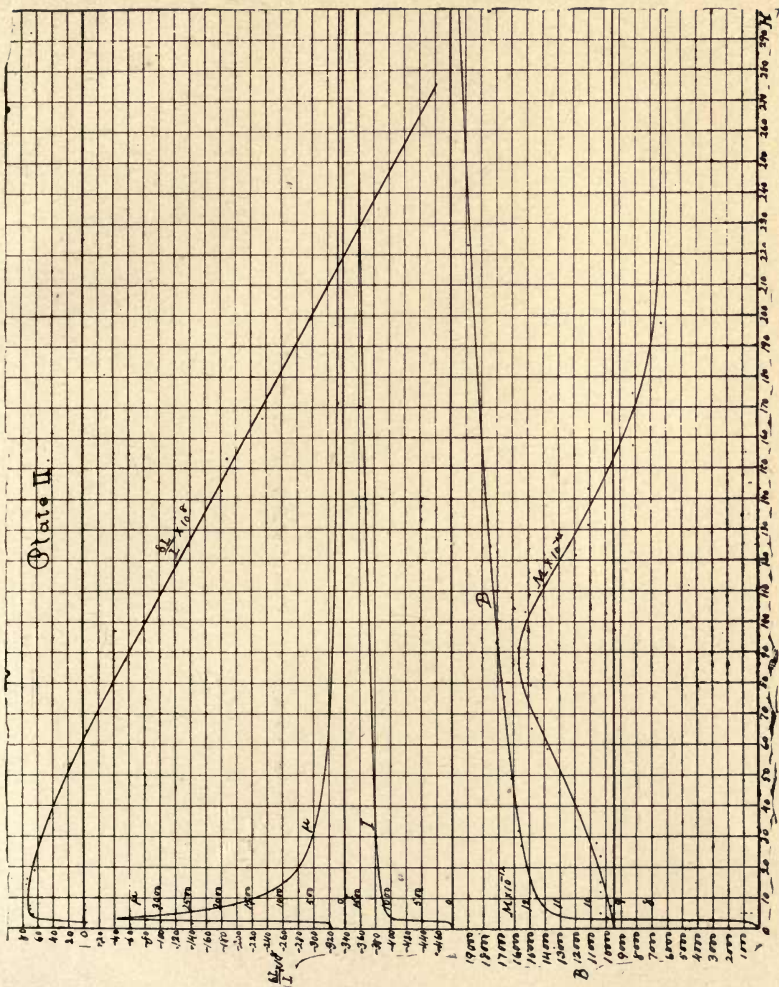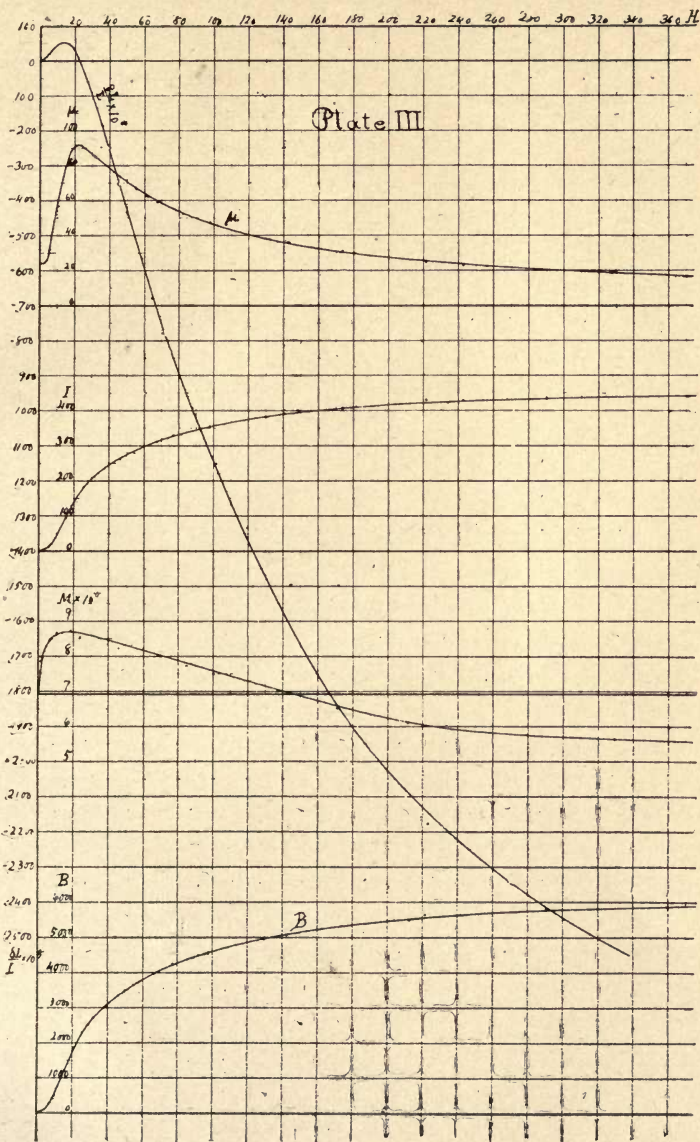
Plate I.

Plate I
(Enlarged)

Plate II

Plate III

Plate IV

Plate V.
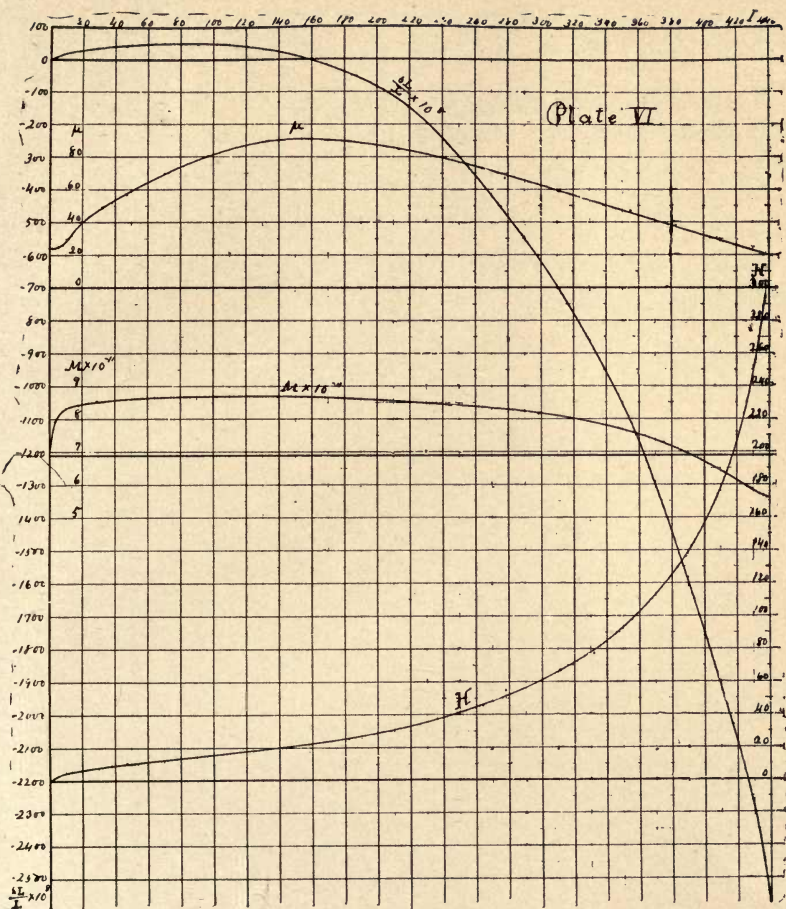
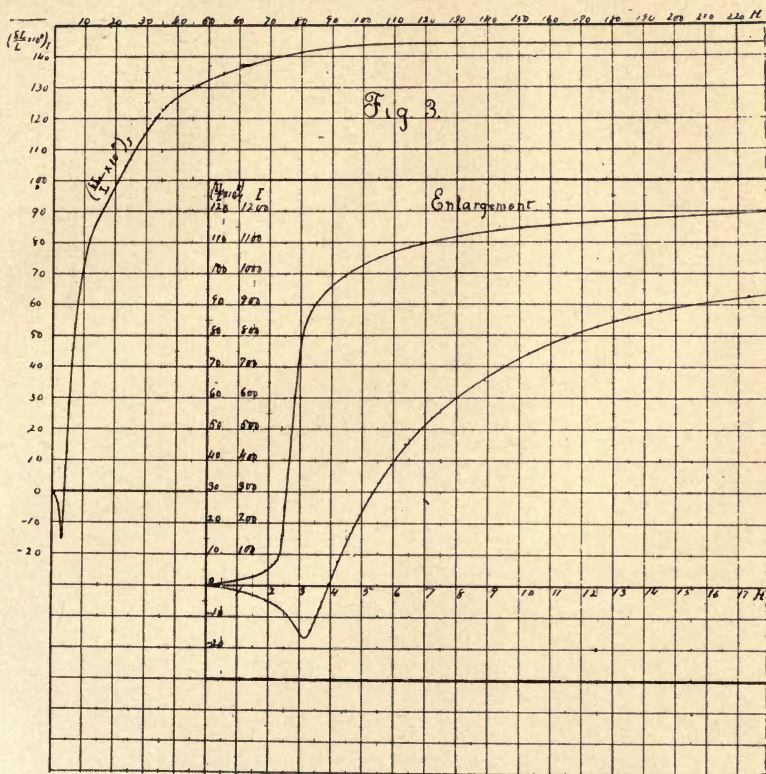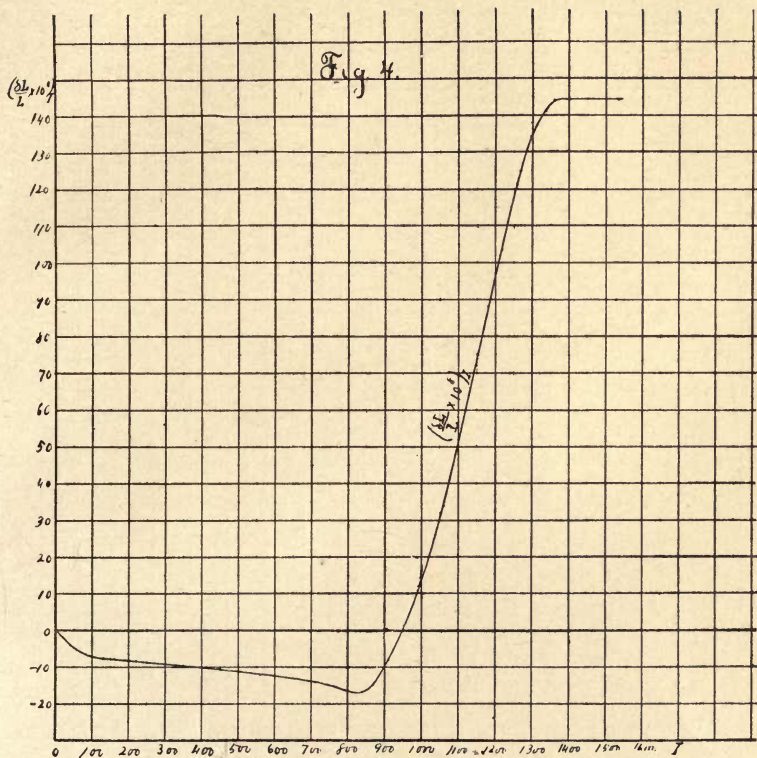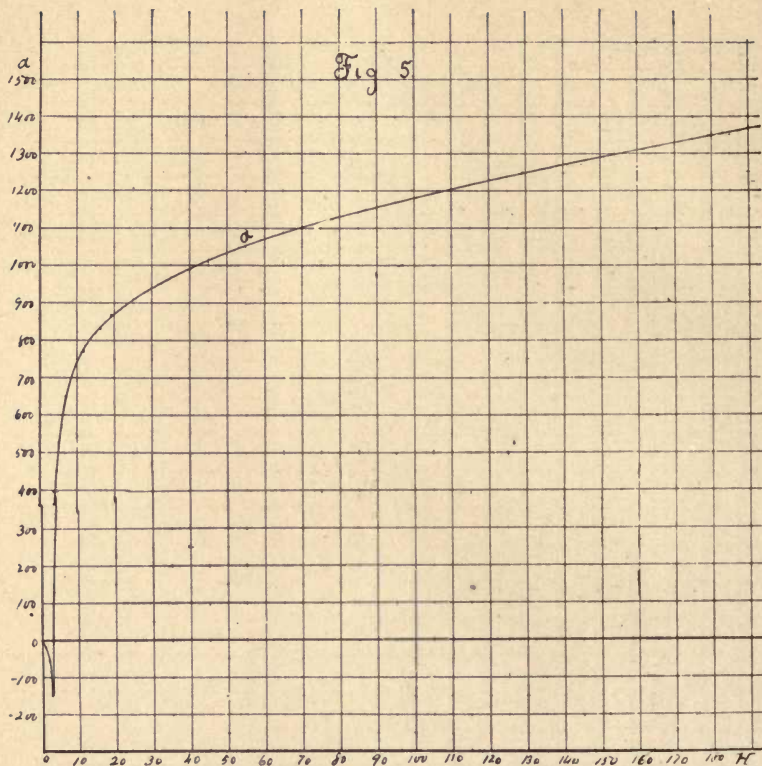Plate VI

Fig 3.

Enlargement

Fig. 4.
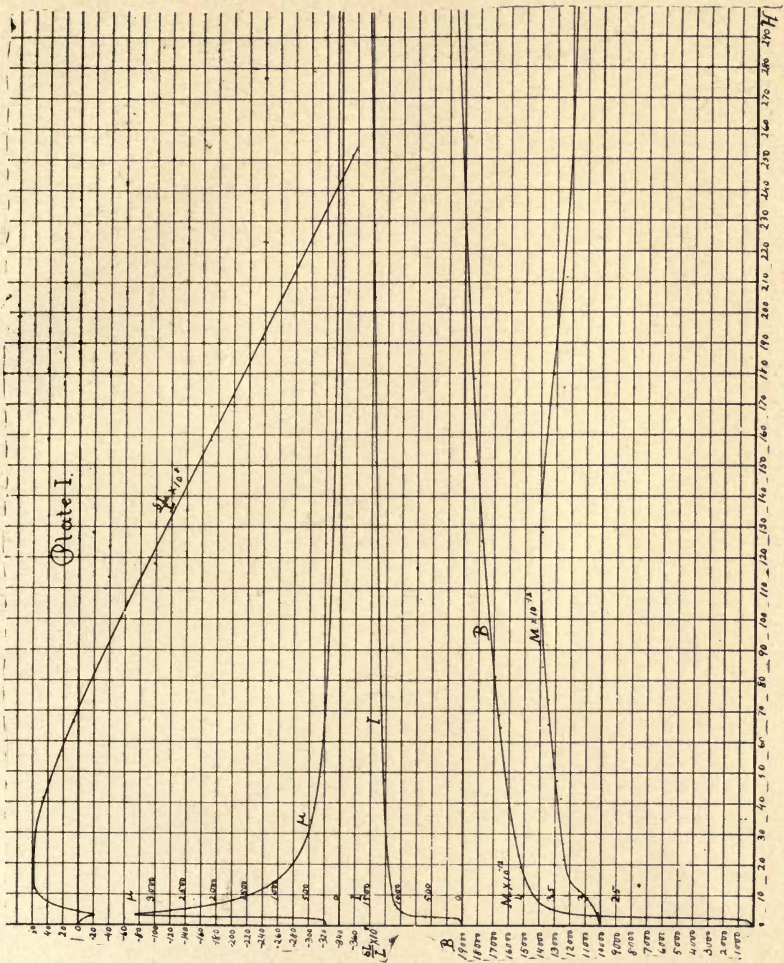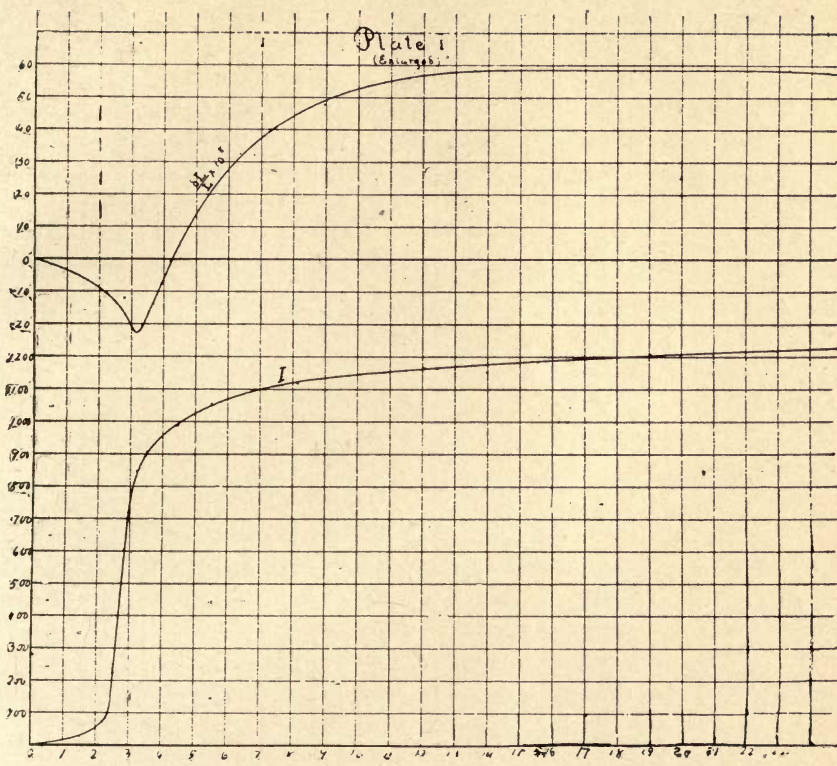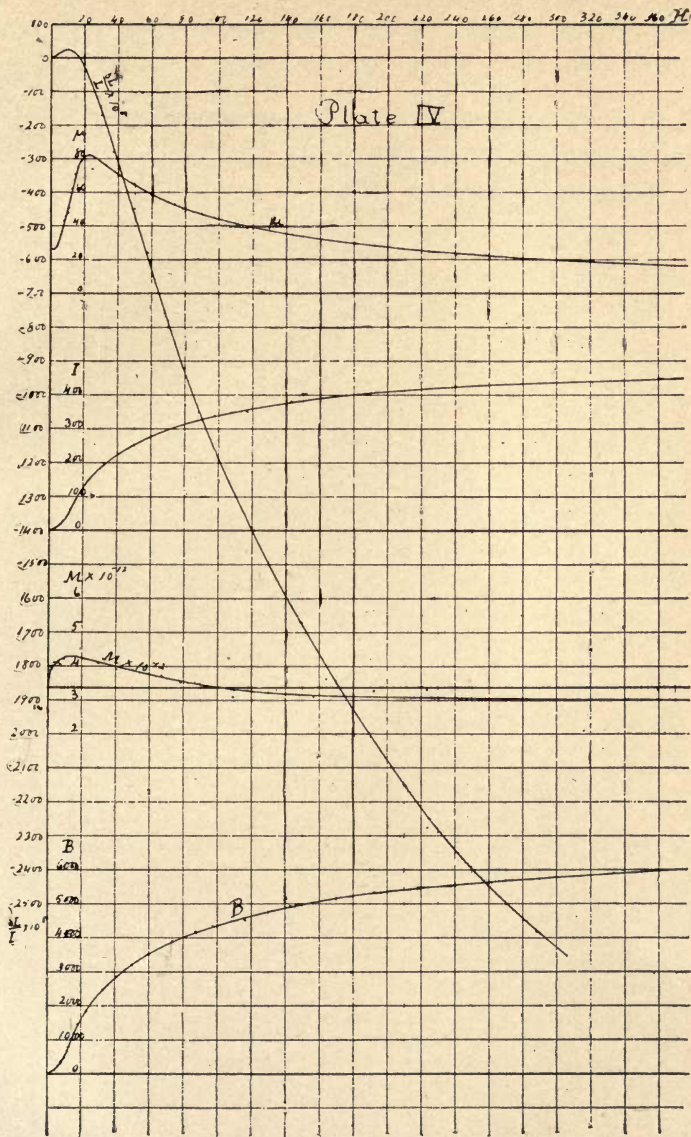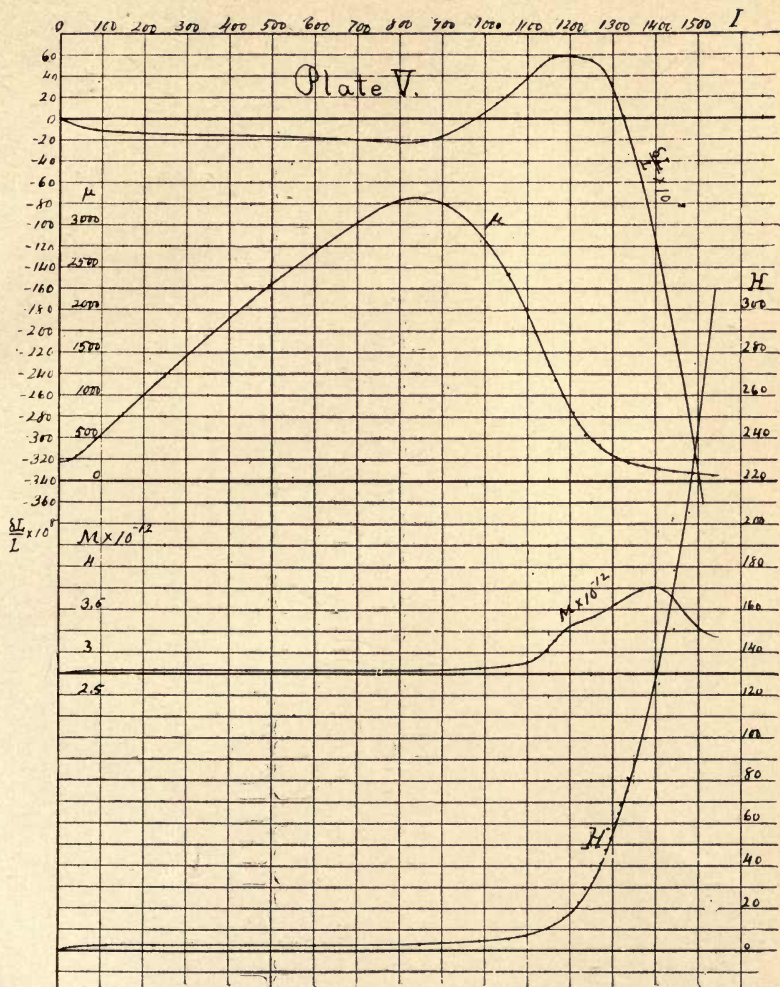
Fig 5

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BIOGRAPHICAL SKETCH.

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